Frequency Distributions of Breaking Load, Tenacity, and Ratio of Cell Wall Thickness to Ribbon Width for Single Cotton Fibers

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Synopsis

Analysis of the frequency distributions of breaking load, tenacity, and ratio of cell wall thickness to ribbon width (C/R) for single cotton fibers in this study has indicated that they can all be represented by β-distributions. This suggests that a parallel model of element configurations exists in which there is a uniform distribution of elemental strengths in single cotton fibers before as well as after slack mercerization. In the case of breaking load and tenacity, the distributions are positively skewed and the skewness decreases on slack mercerization, suggesting that quite a few potentially weak places have been either completely removed or at least strengthened. The changes in C/R ratio on slack mercerization for cottons having a range of maturity have been examined and discussed. The dependence of properties such as breaking load, tenacity, and linear density on cell wall to ribbon width ratio (C/R) have also been studied.

INTRODUCTION

Cotton is a natural textile fiber with a very high degree of single-fiber variability. All the physical properties such as breaking load, tenacity, linear density, maturity, number of reversals, and convolutions, etc., vary widely from fiber to fiber within the same variety, as well as within the same cotton sample. The variability of these parameters has a considerable effect on the performance of a yarn made from these fibers.[1] Consequently, a study of the frequency distributions of some of these parameters, particularly breaking load and tenacity, is very important in order to improve the performance of cottons for better utilization.

From a practical standpoint it is convenient to summarize a large set of single-fiber fracture data, such as breaking load as well as tenacity, by some simple analytical function that has only a few adjustable parameters. An equation is firstly a compact and easily remembered substitute for the data. This is data reduction in the practical sense to diminish the amount of data available. The second reason for fitting equations to scientific data is more fundamental, namely, for model fitting. The equation is then an algebraic representation of a physical model, and the adjustable parameters in it represent the molecular or physical constants. In fitting particular equations to experimental data, the values obtained for the parameters are considered as estimates of the true values of the physical constants. With these objectives we have tried to examine the experimental data on the fracture of single cotton fibers.

While considering the statistics of fracture of real materials, it is assumed that the scatter of results observed in actual physical testing is a consequence of the presence of a distribution of flaws or defects in the sample. According to the

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weak-link theory,[2] failure will occur in each specimen at the location of the most serious flaw. The flaws themselves are rarely visible, and only their effect is observed as a scatter of test results. It was therefore considered necessary to determine and characterize quantitatively this scatter or distribution in the case of fracture of single cotton fibers.

Frequency distribution curves for textile materials have been investigated by various workers in the past.[3–9] Turner[3] has studied the breaking load frequency distribution curves for cotton fibers, yarns, and fabrics. Barratt[4] has given the frequency distribution of cotton fiber length, diameter, breaking load, and extension at break. Duckett et al.[5] have studied frequency distribution of fiber tenacity at different gauge lengths. Clegg[6] has studied the frequency distribution for the breaking loads of single cotton fibers. However, no attempt to fit theoretical frequency curves to the data have been made. Koshal and Turner[7] have studied frequency distribution curves of fiber length, fiber width, convolutions, fiber strength, fiber rigidity, etc. They have tried to fit theoretical curves to their results. They found that a Pearson type I distribution[8] appeared to fit the observed breaking load frequency curves rather well. Somashekar et al.[9,10] at ATIRA have also made an extensive study of the breaking load distribution curves for single fibers. They have studied the effect of maturity,[9] mercerization in the slack and stretched conditions,[10] and the variation of test length on the shape of the breaking load frequency distribution curves.[9,10] They found that in most of the cases a Pearson type I distribution, namely, a β-distribution, was found to explain the observed strength data extremely well, suggesting that in general a uniform distribution of elemental strengths and a parallel model of element configuration exists in single cotton fibers. We have now extended this earlier study of ours to cover the frequency distributions of the breaking load, tenacity, and ratio of cell wall thickness to ribbon width for three cottons differing widely in maturity. Here again it has been found that a β-distribution is able to explain the observed data for all the three parameters rather well.

EXPERIMENTAL

Materials

For the present study of single-fiber breaking load, tenacity and ratio of cell wall thickness to ribbon width (C/R), three cottons namely, Kalyan (*G. herbacium*), Sea Island (*G. barbadense*), and Cotton A (*G. arborium*) covering a range in fineness (180 to 247 mtex) and maturity (65 to 91%) were selected.

Swelling Treatment

These three cottons were slack swollen in 24% (w/w) sodium hydroxide for 30 min at room temperature, washed in water, and air dried.

Test Methods

In order to test the fiber properties of any given cotton, it is necessary to select an appropriate number of fibers which would adequately represent the bulk of the fibers in the sample. In the present study the sampling procedure suggested by Koshal and Turner[7] was used. Small bundles of fibers were taken from different parts of the bulk sample of cotton. These small bundles were then opened up well by hand and a representative sliver was made from these small bundles. This representative sliver was then again divided into small bundles of fibers and drawn once more. Employing this procedure the fibers in the sliver were mixed thoroughly. In order to select individual fibers from the representative sliver, small tufts were taken at random, and again single fibers were picked up randomly from these small tufts.

Each single fiber thus selected from the representative sliver was then mounted between two aluminum strips. One strip was about 5 cm long, and the other strip was about 1.5 cm long. The weights of the shorter strips were kept constant, about 150 mg. Each pair of strips was mounted on a wooden stand in such a way that the distance between the two strips was 1 cm. The single fibers were fixed on these strips using a strong quick drying adhesive. The stand with the strips and mounted fibers were kept for 24 hr in a room maintained at 65% R.H. and 27°C.

For measurement of linear density the strips were transferred to a Vibroscope. The standard procedure[11] was used for measuring the linear density. The length l of the fiber was first measured precisely, and then the fiber was brought between two electrodes. The frequency applied between two electrodes was varied, and the frequency f at which the amplitude was maximum was determined. The linear density m was then calculated using the following formula:

$$m = \frac{980\,M}{4l^2 f^2} \tag{1}$$

where M is the mass of the lower strip in grams and f is the resonant frequency for maximum amplitude. For tensile strength determination the same fiber was transferred to an Instron tensile tester. The fiber was broken employing a cross-head speed of 0.5 cm/min.

After rupture of the fiber the strips were removed from the Instron jaws, and the broken ends of the fibers were cut from the strips and mounted on a glass slide. Liquid paraffin was used as the embedding medium. The slide then was transferred to the stage of a projection microscope. Ribbon width R and lumen width L were measured at the same place selected between two successive convolutions. Five measurements, including one near the break, were made at different places along the test length. Averages of these five readings for ribbon width R and lumen width L were calculated. The mean cell wall thickness was then calculated using the following formula:

$$C = (R - L)/2 \tag{2}$$

In order to calculate average ratio of cell wall thickness to ribbon width, C/R, the average value of C calculated using eq. (2) was divided by the average value of R. Three hundred fibers were examined in a similar manner for each cotton sample.

For analysis the observed tensile strength data were grouped in such a way that the total range of breaking load or tenacity was divided into a number of small class intervals and the frequency (number of fibers) in each interval was determined. From these data the observed histograms were drawn. Theoretical frequency distribution curves having the β-type density function[9] given by

$$Y = Kx^{m-1}(a - x)^{n-1} \tag{3}$$

were then fitted to the observed data. In the above β-distribution, Y is the frequency of occurrence of the variable x (which can be the breaking load, the tenacity, or the C/R ratio), K is a normalizing constant, a is the total range of the frequency curve, and m and n are parameters governing the asymmetrical shape of the frequency curve. The parameters m and n were estimated from the observed frequency data by the method of moments, while the range a was estimated by the minimum chi-square (χ^2) method.

RESULTS AND DISCUSSION

The physical and mechanical properties of the cottons studied are given in Table I. Data shown under columns 1 to 7 are taken from another study,[12] whereas data shown under columns 8 to 18 are the averages for 300 single-fiber measurements presented in this study. It is known that linear density obtained from vibroscope measurements is not very accurate.[13] Consequently, to check the validity of these results a comparison with gravimetric fineness has been made. The ratio of fineness measured with the vibroscope to gravimetric fineness (m_{vib}/m_{gra}) has been calculated and is given in Table I. From the table it is seen that results obtained from the vibroscopic method are slightly lower than those obtained from gravimetric fineness, and the ratio of the two quantities is in the range of 0.88 to 0.96. The considerably high ratio suggests that the results are reliable within experimental error.

Analysis of Breaking Load Frequency Distribution for Raw and Mercerized Cottons

The data for 300 fibers were grouped in small equal class intervals of breaking load. Care was taken to have at least 9 to 10 class intervals in the total range for each cotton. The number of fibers breaking in each class interval was determined and is given as the observed frequency in Tables II(a) and II(b). The class intervals were not the same for each cotton, hence the midpoint of the interval is given in each case. The theoretical frequencies have been calculated from eq. (3) using a curve-fitting procedure to obtain the best fit. The observed and theoretical frequencies for the raw cottons are shown in Table II(a) and for mercerized cottons, in Table II(b). In Figures 1(a) and 1(b) the observed frequencies are shown as histograms and all the theoretical frequencies (β-distribution), as smooth curves. The values for the parameters m, n, a and χ^2 (the goodness of fit) are shown in Table II(c).

From Figure 1(a) it is seen that the frequency distribution of the breaking load for raw cottons is positively skewed. The skewness decreases with increasing maturity. From Table II(c) it is seen that for raw cottons the parameter m is almost constant, whereas n has a tendency to decrease as maturity increases. This may be due to the positive skewness factor of the observed frequency curves.[9] The range a for these three cottons is not very different. Since the goodness of fit (χ^2) values are very low, it can be confidently assumed that the distribution of the breaking load of single cotton fibers conforms to a β-distribution. Now since a parallel model of element configurations leads to a β-distribution of breaking stresses, if the underlying distribution of the elements is assumed to be uniform,[14] we can conclude from this study that such a distribution of elements exists in single cotton fibers.

TABLE I
Physical and Mechanical Properties of Raw and Mercerized Cottons

Cotton	Gravimetric fineness, μg/cm	Maturity, %	Perimeter, μm	Cell wall thickness, μm	Total area of cross section, μm²	Area of secondary wall, μm²	Circularity, δ	Single fiber data: Breaking load Mean, g	Breaking load S.D.	Breaking load C.V., %	Tenacity Mean, g/tex	Tenacity S.D.	Tenacity C.V., %	Linear density Mean, mtex	Linear density C.V., %	C/R Mean	C/R S.D.	C/R C.V., %	m_{vib}/m_{gra}
Kalyan (raw)	2.47	65	58.8	4.4	180.6	168.7	0.66	3.89	1.77	46	18.1	6.7	37	218	35	0.26	0.080	33	0.88
Sea Island (raw)	1.81	78	49.4	4.2	144.7	134.5	0.75	4.53	2.31	51	28.3	10.9	38	159	33	0.25	0.082	32	0.88
Cotton A (raw)	1.89	91	48.1	4.5	153.4	145.0	0.83	4.76	2.20	46	27.2	11.1	41	175	27	0.29	0.081	28	0.93
Kalyan (slack mercerized)	—	—	—	6.4	—	—	—	4.86	1.95	40	18.1	6.3	35	275	29	0.38	0.076	20	—
Sea Island (slack mercerized)	2.22	—	—	5.6	—	—	—	4.56	2.02	44	23.4	9.0	38	198	30	0.42	0.073	18	0.89
Cotton A (slack mercerized)	2.14	—	—	6.6	—	—	—	5.52	2.17	39	27.5	10.3	38	205	24	0.43	0.049	11	0.96

TABLE II(a)
Breaking Load Frequency Distribution of Raw Cottons

Kalyan			Sea Island			Cotton A		
Midpoint of class interval, g	Frequency		Midpoint of class interval, g	Frequency		Midpoint of class interval, g	Frequency	
	Obs.	Calc.		Obs.	Calc.		Obs.	Calc.
0.5	8	7	0.75	27	29	0.7	11	12
1.5	42	40	2.25	67	67	2.1	57	51
2.5	51	63	3.75	71	73	3.5	57	71
3.5	75	65	5.25	57	61	4.9	77	68
4.5	56	53	6.75	46	41	6.3	51	51
5.5	29	36	8.25	21	23	7.7	30	30
6.5	22	21	9.75	13	9	9.1	15	14
7.5	11	10	—	—	—	10.5	3	4
8.5	4	4	—	—	—	11.9	1	1
9.5	2	1	—	—	—	—	—	—

TABLE II(b)
Breaking Load Frequency Distribution of Mercerized Cottons

Kalyan			Sea Island			Cotton A		
Midpoint of class interval, g	Frequency		Midpoint of class interval, g	Frequency		Midpoint of class interval, g	Frequency	
	Obs.	Calc.		Obs.	Calc.		Obs.	Calc.
0.5	0	1	0.5	0	4	0.7	1	3
1.5	15	14	1.5	34	24	2.1	30	27
2.5	42	39	2.5	45	43	3.5	58	55
3.5	53	57	3.5	37	52	4.9	60	68
4.5	59	59	4.5	51	52	6.3	60	61
5.5	41	50	5.5	54	44	7.7	48	42
6.5	46	35	6.5	29	33	9.1	19	21
7.5	18	21	7.5	28	21	10.5	6	7
8.5	12	11	8.5	6	11	11.9	1	1
9.5	3	5	9.5	3	4	—	—	—
10.5	4	2	10.5	2	1	—	—	—

The effect of slack mercerization on the breaking load frequency distribution is seen from Table II(b) and Figure 1(b). It will be noticed that on mercerization the skewness decreases but still remains different for cottons of different maturities. Mature cottons have more symmetrical breaking load distributions than immature ones even after mercerization. From Table II(c) it can be seen that just as in the case of raw cottons the parameter m appears to be constant, whereas n decreases as maturity increases. Here again, the χ^2 values are low, suggesting that even after mercerization the breaking load distributions can still be very well represented by β-distribution curves.

On mercerization the curves become less skewed and the mode moves toward higher breaking loads. This confirms earlier findings that on mercerization in general weak places are either substantially strengthened or perhaps some are even completely removed. Some weak places may be retained after mercerization, but on the whole the strength of the cotton is improved.

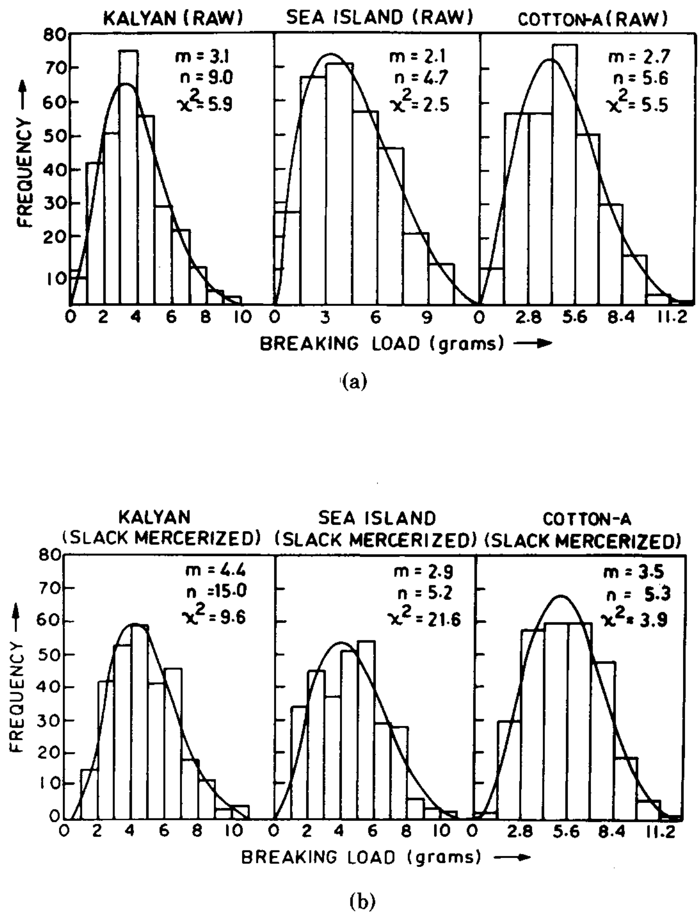


Fig. 1. (a) Breaking load frequency distributions for Kalyan, Sea Island, and cotton A raw cottons. (b) Breaking load frequency distributions for Kalyan, Sea Island, and cotton A after slack mercerization.

Analysis of Frequency Distributions for Tenacity of Raw and Mercerized Cottons

The tenacity was calculated accurately for each fiber since the proper linear density was determined for each fiber from vibroscopy. The data for 300 fibers were grouped into small equal class intervals of tenacity. The theoretical frequencies were calculated from the observed frequencies using eq. (3). The observed and theoretical frequencies for raw cottons are given in Table III(a) and for slack mercerized, in Table III(b). The data are plotted in Figures 2(a) and 2(b), respectively. The observed frequencies are shown as histograms, while the theoretical frequencies are represented by smooth curves. Here again, as in the breaking load frequency distribution, the tenacity frequency distributions are slightly positively skewed and the skewness has a tendency to decrease as maturity increases.

The effect of mercerization on the tenacity frequency distribution curves of

TABLE II(c)
Parameters for Different Frequency Distribution Curves

Cotton	Fineness, $\mu g/cm$	Total area of cross section, μm^2	Area of secondary wall, μm^2	Breaking load				Tenacity				C/R			
				m	n	a	χ^2	m	n	a	χ^2	m	n	a	χ^2
Kalyan (Raw)	2.47	180.6	168.7	3.1	9.0	15.0	5.9	4.6	10.6	60.0	2.6	4.1	3.3	0.47	6.1
Sea Island (Raw)	1.81	144.7	134.5	2.1	4.7	14.2	2.5	3.6	4.9	67.0	5.1	3.6	3.2	0.46	7.8
Cotton A (Raw)	1.89	153.4	145.0	2.7	5.6	14.4	5.5	3.4	5.9	75.0	5.7	4.7	3.4	0.50	10.5
Kalyan (slack mercerized)	—	—	—	4.4	15.0	21.1	9.6	4.3	6.6	46.0	4.6	5.7	1.9	0.50	5.7
Sea Island (slack mercerized)	2.22	—	—	2.9	5.2	12.7	21.6	3.5	5.1	57.6	8.7	5.2	1.1	0.50	12.8
Cotton A (slack mercerized)	2.14	—	—	3.5	5.3	13.9	3.9	3.4	4.5	63.0	8.0	10.6	1.8	0.50	15.8

TABLE III(a)
Tenacity Frequency Distribution of Raw Cottons

Kalyan			Sea Island			Cotton A		
Midpoint of class interval, g/tex	Frequency Obs.	Frequency Calc.	Midpoint of class interval, g/tex	Frequency Obs.	Frequency Calc.	Midpoint of class interval, g/tex	Frequency Obs.	Frequency Calc.
2	0	1	3	2	1	3	0	2
6	17	15	9	18	15	9	25	21
10	39	43	15	29	38	15	47	45
14	67	64	21	61	56	21	52	59
18	67	66	27	59	62	27	65	60
22	51	52	33	62	56	33	45	50
26	28	32	39	38	40	39	35	35
30	19	16	45	22	23	45	20	20
34	7	6	51	8	9	51	11	9
38	2	2	57	3	2	57	2	3

TABLE III(b)
Tenacity Frequency Distribution of Mercerized Cottons

Kalyan			Sea Island			Cotton A		
Midpoint of class interval, g/tex	Frequency Obs.	Frequency Calc.	Midpoint of class interval, g/tex	Frequency Obs.	Frequency Calc.	Midpoint of class interval, g/tex	Frequency Obs.	Frequency Calc.
2	1	1	3	4	3	3	1	2
6	10	14	9	32	27	9	20	17
10	47	40	15	42	56	15	41	39
14	63	62	21	74	70	21	42	55
18	57	67	27	70	63	27	61	59
22	60	55	33	45	43	33	63	51
26	33	34	39	14	21	39	30	36
30	17	16	45	7	6	45	19	19
34	5	5	51	1	1	51	5	6
—	—	—	—	—	—	57	1	1

Kalyan and cotton A is not much, but in case of Sea Island the skewness has become more positive. This may be explained on the basis that on mercerization the average breaking load for Kalyan and Cotton A has increased considerably and at the same time the average linear density has also increased. As a result the average tenacity values are not changed much (see Table I). But in the case of Sea Island the average breaking load has increased slightly on mercerization, while the average linear density has increased to a greater extent. Hence, the average tenacity has decreased. As a result the mode in the distribution has shifted slightly to lower tenacity values resulting in higher positive skewness. This suggests that Sea Island, which belongs to the *G. barbadense* species, differs from the other two cottons belonging to the *G. herbacium* and *G. arborium* species.

The parameters m, n, a, and χ^2 are given in Table II(c). It can be seen that for both raw and mercerized cottons m remains almost constant, n has a tendency to decrease as maturity increases, and a, the range, increases with maturity. The

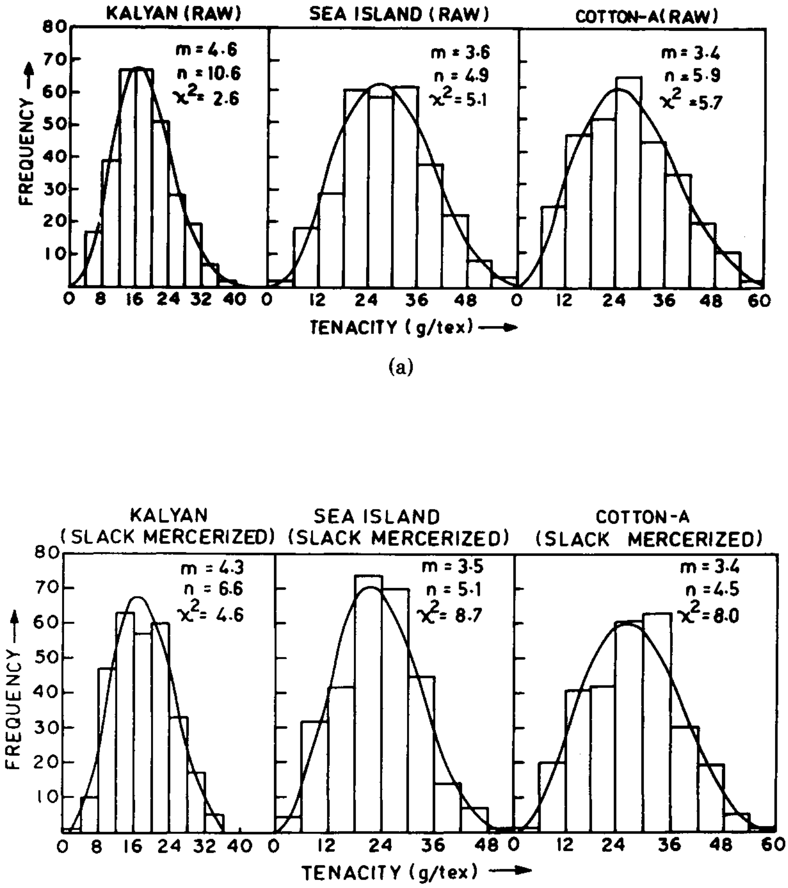


Fig. 2. (a) Tenacity frequency distributions for Kalyan, Sea Island, and cotton A raw cottons. (b) Tenacity frequency distributions for Kalyan, Sea Island, and cotton A after slack mercerization.

range a for mercerized cottons is less than the corresponding values for raw cottons. Here again the goodness of fit (χ^2) values are very low, and consequently it can be confidently assumed that the tenacity distribution of single cotton fibers can be represented by a β-distribution.

Relationship Between Frequency Distribution Parameters and Other Physical Properties

Attempts have been made to try to correlate the distribution parameters m and n given in Table II(c) with some of the physical properties of the cotton fibers. It has been found that there is perhaps some relationship between these parameters obtained for the breaking load and breaking tenacity distributions and the total area of the fiber cross sections, the area of the secondary wall, and the fineness. Figure 3 shows the linear relationships observed in this study. It can be seen in general that as the total cross-sectional area, the area of secondary wall, or the fineness increase, the values of m and n both increase in the case of

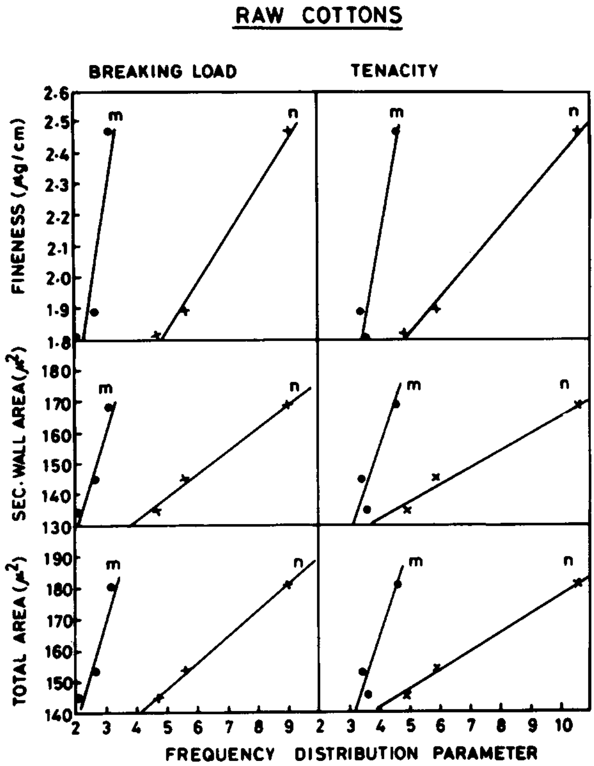


Fig. 3. Relationship between β-distribution parameters m and n and physical characteristics such as total cross-section area, secondary wall area, and fineness.

the breaking load and tenacity distributions of the untreated fiber control samples. In all cases, while m does not change very rapidly, n undergoes considerable change with an increase in any of these physical properties. Thus, the coarser cottons have distributions of strength which are more peaked and more asymmetrical than their finer counterparts. The deviations from symmetry, when $m = n$, are more for coarse cotton fibers. However, it must be admitted that these observations are merely qualitative and will have to be confirmed after a study of many more cotton samples. The present study merely establishes the possibility of such a trend.

Analysis of Frequency Distribution of *C/R* Ratio for Raw and Mercerized Cottons

The data for the three cottons were grouped in equal class intervals of the C/R ratio. Here, the minimum value is zero and the maximum value is 0.50 (total range of C/R). The theoretical frequencies were calculated from the observed data using eq. (3). The observed and theoretical frequencies are given in Tables IV(a) and IV(b) and plotted in Figures. 4(a) and 4(b) for raw and mercerized cottons, respectively. The observed frequencies are shown as histograms and the theoretical frequencies as smooth curves. Values of parameters m, n, a, and χ^2 are given in Table II(c).

From Figure 4(a) it can be seen that the frequency distributions for raw cottons

TABLE IV(a)
C/R Ratio Frequency Distribution of Raw Cottons

Kalyan			Sea Island			Cotton A		
Midpoint of class interval	Frequency Obs.	Frequency Calc.	Midpoint of class interval	Frequency Obs.	Frequency Calc.	Midpoint of class interval	Frequency Obs.	Frequency Calc.
0.025	0	0	0.025	0	1	0.025	0	0
0.075	7	6	0.075	9	11	0.075	3	2
0.125	19	22	0.125	39	30	0.125	17	12
0.175	50	42	0.175	43	50	0.175	25	29
0.225	51	60	0.225	58	63	0.225	39	50
0.275	65	66	0.275	73	63	0.275	66	66
0.325	66	57	0.325	44	50	0.325	83	67
0.375	28	34	0.375	30	28	0.375	47	50
0.425	10	9	0.425	6	7	0.425	19	23
—	—	—	—	—	—	0.475	2	2

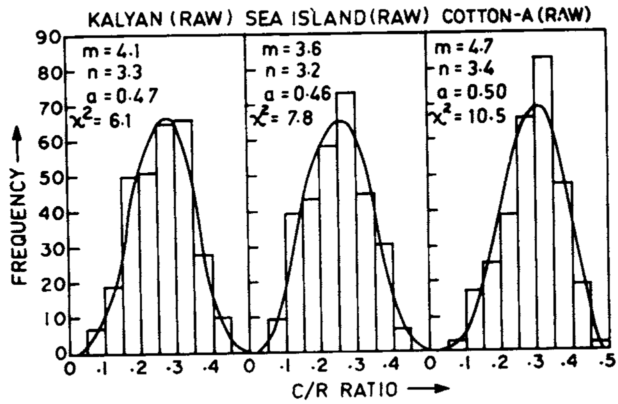
TABLE IV(b)
C/R Ratio Frequency Distribution of Mercerized Cottons

Kalyan			Sea Island			Cotton A		
Midpoint of class interval	Frequency Obs.	Frequency Calc.	Midpoint of class interval	Frequency Obs.	Frequency Calc.	Midpoint of class interval	Frequency Obs.	Frequency Calc.
0.12	1	1	0.16	3	1	0.31	8	4
0.16	3	2	0.20	4	3	0.33	9	7
0.20	6	6	0.24	9	6	0.35	7	11
0.24	10	12	0.28	8	12	0.37	22	16
0.28	19	22	0.32	13	20	0.39	32	24
0.32	44	35	0.36	29	33	0.41	27	33
0.36	43	49	0.40	54	49	0.43	33	43
0.40	57	61	0.44	85	71	0.45	48	[illegible]
0.44	70	63	0.48	84	94	0.47	53	55
0.48	40	43	—	—	—	0.49	44	39

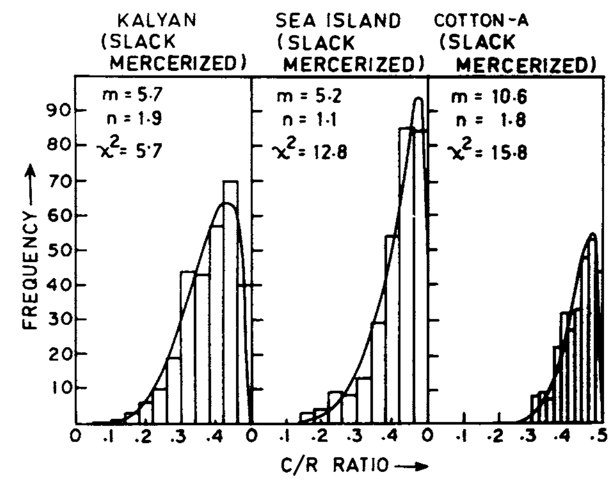
are slightly negatively skewed. Breaking load distribution curves are slightly positively skewed, whereas C/R ratio distribution curves for the same fibers are slightly negatively skewed. Contrary to the previous two strength distributions, m is greater than n, and in this case n remains almost constant while m has a tendency to change with maturity. This may be due to the negative skewness of the observed frequency curves.

The effect of mercerization on the frequency distribution curves is very clearly seen in Figure 4(b). On mercerization the distributions become highly negatively skewed, and naturally the three parameters m, n, and a change, see Table II(c). As a result, m becomes larger whereas n becomes smaller. The range a goes to the higher limit of 0.50 but does not start from zero. The minimum threshold value is thus much higher, particularly in the case of cotton A. Here again goodness of fit (χ^2) values are low, which confirms that the C/R ratio distribution can be very well represented by a β-distribution in practice.

The effect of mercerization on the frequency distribution of the C/R ratio is clearly seen from Figures 5(a), 5(b), and 5(c). In these figures solid lines show the calculated distributions for raw cottons, and the broken lines show the cal-



(a)

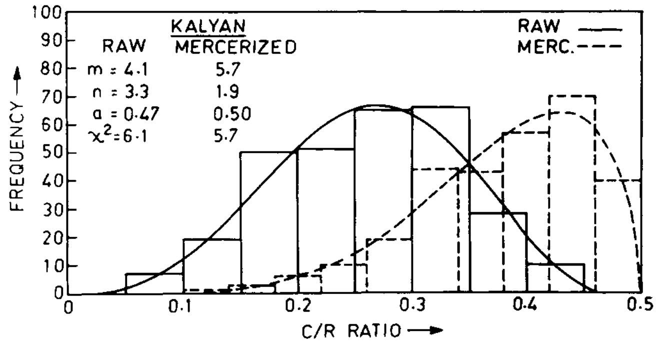


(b)

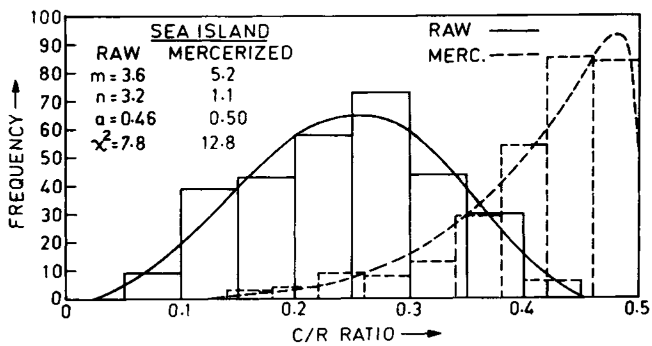
Fig. 4. (a) Frequency distributions of C/R ratio for Kalyan, Sea Island, and cotton A raw cottons. (b) Frequency distributions of C/R ratio for Kalyan, Sea Island, and cotton A after slack mercerization.

culated distributions for the same cottons after slack mercerization. It can be seen that the curves have in general shifted to higher C/R ratio values, and as a result the modes have moved to much higher values. The shift of the curves is possibly due to two reasons: (i) fiber shape becomes more circular on mercerization and as a result the ribbon width (R) decreases, and (ii) cell wall thickness (C) increases due to slack swelling. Changes in both these factors result in higher C/R ratios for mercerized cottons.

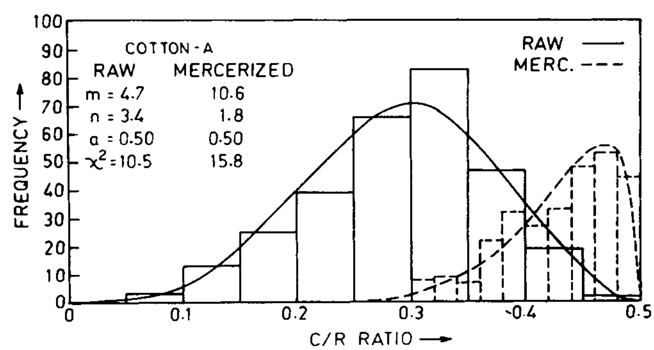
The shifts in the overall distribution curve as well as the increased negative skewness for the mercerized cotton suggest that in any cotton variety the immature fibers swell more than mature fibers. When the three curves are compared, one interesting feature is noticed. It can be seen that the tails of the



(a)



(b)



(c)

Fig. 5. (a) Frequency distributions of C/R ratio for Kalyan cotton, raw as well as after slack mercerization. (b) Frequency distributions of C/R ratio for Sea Island cotton, raw as well as after slack mercerization. (c) Frequency distributions of C/R ratio for cotton A, raw as well as after slack mercerization.

distributions on the low C/R value side are different for the three mercerized cottons having three different initial maturities for their untreated control cottons. In the case of Kalyan cotton, with an initial low maturity, the range extends down to 0.10 in C/R; while for cotton A, with an initial high maturity, it is greater than 0.25 in C/R ratio. Sea Island cotton, with an intermediate maturity, lies between the other two cottons. For all the three mercerized cottons the maximum value of C/R is 0.50 and is the upper limit of the distribution.

From these results it appears that the change in the distribution of C/R ratio after slack mercerization will be greater the higher the initial maturity of the cotton sample.

Dependence of Breaking Load, Tenacity, and Linear Density on Cell Wall Thickness to Ribbon Width Ratio

The strength of a cotton fiber is to a large extent determined by its fibrillar orientation, linear density, and maturity. It is also well known that cellulose in the secondary wall of a fiber is laid down in the form of layers. The extent of deposition of these layers is not uniform, even in fibers on a single seed, and results in a wide range of maturity. Many workers have studied the effect of cell wall thickness on the strength of a fiber. Clegg[6] found no direct relationship between cell wall thickness and strength and concluded that in addition to the abnormalities of hair weight and wall thickness there are possible abnormalities in the composition of the fiber.

The structure of a cotton fiber is well known. The smallest unit of the structure is a crystalline elementary fibril approximately 30 to 60 Å wide and made up of individual parallel chains of cellulose.[15] These elementary fibrils coalesce along their length, and this aggregation appears as a series of concentric fibrillar aggregates or lamellae. Longitudinally, the fibrils spiral about the fiber axis at angles of 20 to 30 degrees with frequent reversals of direction of the spiral. It is known that the spiral angle varies across the width of a fiber and decreases from the periphery to the core. This difference in the orientation of fibrils results in slippage of the planes during tensile strain[16] and hence may affect the final breaking strength of a fiber.

The other important factor is the distribution of pores or voids across the width of the fiber within the secondary wall. So far, very little is known about this. Peterlin and Ingram[17] have studied this aspect and have raised some doubt that spacing may vary across the width of the fiber. In the present study we have examined the variation of breaking load, tenacity, and linear density as a function of C/R ratio for three cottons differeing fairly widely in maturity. The effect of slack mercerization on these various interrelationships has also been studied.

The data presented earlier in Tables II and III for these three cottons has been used here after suitable grouping in equal class intervals in the C/R ratio. The averages in each group or class interval in C/R were calculated for all the parameters and are given in Tables V, VI, and VII. These data are plotted in Figures 6, 7, and 8.

TABLE V
Data Grouped in Equal Class Intervals in C/R Ratio

Cotton	C/R ratio range	Average C/R ratio	Frequency	Breaking load, g	Tenacity, g/tex	Linear density, mtex
Kalyan (raw)	0.051–0.100	0.08	7	1.46	13.6	102
	0.101–0.150	0.13	19	2.27	16.1	136
	0.151–0.200	0.18	50	2.80	16.8	166
	0.201–0.250	0.23	51	3.78	18.4	208
	0.251–0.300	0.27	65	4.05	17.9	231
	0.301–0.350	0.32	66	4.90	19.4	257
	0.351–0.400	0.37	28	4.71	17.9	272
	0.401–0.450	0.42	10	4.30	17.9	252
Kalyan (mercerized)	0.101–0.140	0.11	1	1.90	13.3	143
	0.141–0.180	0.15	3	3.14	17.4	174
	0.181–0.220	0.20	6	3.25	16.9	222
	0.221–0.260	0.23	10	2.97	17.1	175
	0.261–0.300	0.28	19	4.22	18.4	226
	0.301–0.340	0.32	44	4.79	18.4	265
	0.341–0.380	0.37	43	4.90	18.3	274
	0.381–0.420	0.40	57	5.19	18.2	293
	0.421–0.460	0.44	70	5.24	18.0	299
	0.461–0.500	0.48	40	4.96	17.6	289

TABLE VI
Data Grouped in Equal Class Interval in C/R Ratio

Cotton	C/R ratio range	Average C/R ratio	Frequency	Breaking load, g	Tenacity, g/tex	Linear density, mtex
Sea Island (raw)	0.051–0.100	0.09	9	2.25	22.4	86
	0.101–0.150	0.12	39	2.54	25.5	95
	0.151–0.200	0.17	43	3.76	27.3	135
	0.201–0.250	0.23	58	4.72	30.3	156
	0.251–0.300	0.28	73	5.51	29.9	182
	0.301–0.350	0.32	44	5.28	28.5	185
	0.351–0.400	0.37	30	5.16	24.6	201
	0.401–0.450	0.42	6	4.28	19.1	219
Sea Island (mercerized)	0.141–0.180	0.17	3	2.02	23.7	86
	0.181–0.220	0.20	4	2.29	22.9	88
	0.221–0.260	0.24	9	2.94	24.4	113
	0.261–0.300	0.28	8	3.74	23.9	151
	0.301–0.340	0.32	13	4.77	26.1	152
	0.341–0.380	0.36	29	4.09	22.3	186
	0.381–0.420	0.40	54	5.27	26.2	205
	0.421–0.460	0.44	85	5.15	23.7	219
	0.461–0.500	0.49	84	4.08	20.2	205

Dependence of Breaking Load on C/R Ratio for Raw and Slack Mercerized Cottons

It can be seen from Figure 6(a) that the breaking load for raw cottons initially increases with *C/R* ratio, reaches a maximum between *C/R* of 0.30 and 0.40, and at higher values the breaking load tends to decrease slightly for all cottons. There

TABLE VII
Data Grouped in Equal Class Intervals in C/R Ratio

Cotton	C/R ratio range	Average C/R ratio	Frequency	Breaking load, g	Tenacity, g/tex	Linear density, mtex
Cotton A (raw)	0.051–0.100	0.09	3	1.97	16.6	113
	0.101–0.150	0.13	17	2.90	24.8	119
	0.151–0.200	0.18	25	3.78	26.8	136
	0.201–0.250	0.23	39	4.45	29.2	153
	0.251–0.300	0.28	66	4.68	27.5	177
	0.301–0.350	0.33	83	5.02	26.6	191
	0.351–0.400	0.38	47	5.49	27.8	201
	0.401–0.450	0.43	19	5.52	27.9	202
	0.451–0.500	0.46	2	4.35	22.0	195
Cotton A (mercerized)	0.301–0.320	0.31	8	4.35	26.6	172
	0.321–0.340	0.33	9	5.36	26.9	187
	0.341–0.360	0.35	7	6.13	32.6	178
	0.361–0.380	0.37	22	5.92	31.4	182
	0.381–0.400	0.39	32	6.50	33.2	202
	0.401–0.420	0.41	27	6.16	32.3	197
	0.421–0.440	0.43	33	5.84	29.3	201
	0.441–0.460	0.45	48	6.30	28.7	219
	0.461–0.480	0.47	53	4.81	23.4	210
	0.481–0.500	0.49	44	4.26	20.4	216

does not appear to be much difference in the shape of the curves with percent maturity determined by the standard method for maturity determination.

Figure 6(b) shows the dependence of breaking load on C/R ratio for the same cottons after slack mercerization. Here, it will be noticed that the curves have shifted in general to higher values of C/R and the average slope also seems to be higher for higher values of the average maturity of the untreated control cotton. The fall in breaking load at higher C/R values is also more pronounced for higher average maturity cottons.

The shift in the curves toward higher C/R values is probably due to the increase in cell wall thickness and a decrease in the ribbon width due to increased circularity of the cross-section after mercerization. The combined effect is to increase the C/R considerably after slack mercerization.

The leveling off of breaking load at high maturity has been reported by many workers.[6,13,18] Berkley[18] has observed that the tensile strength of the fiber increased for 12 to 18 days after secondary thickening was initiated and reached to maximum at almost the 35th day after flowering, or three to four weeks before the bolls opened. No appreciable increase or change was observed in the strength after this period. Clegg[6] observed a linear relationship between tensile strength and cell wall thickness at rupture after removal of abnormal fibers. However, such a linear relationship was found only for two cottons. From this result it was concluded that in addition to the abnormalities of hair weight and wall thickness there were possible abnormalities in the composition of the wall of the hair. Patel and Patil[13] found that breaking load initially increased with linear density and then tended to level off. These variations were clearly shown by plotting tenacity against linear density.

In our study we have measured C/R at five different places on the fiber length

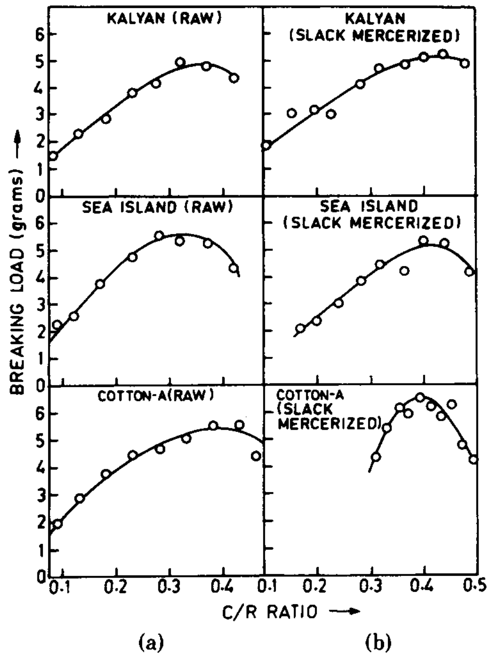


Fig. 6. Relationship between breaking load and *C/R* ratio for Kalyan, Sea Island, and cotton A: (a) raw cottons; (b) after slack mercerization.

tested. That means that it is an average value over 1 cm in fiber length. It is known that strength is a single point phenomenon, i.e., strength of a test specimen is that of its weakest element in that test length. Consequently, the fall in breaking load at high *C/R* values may be due to some extent to thick fibers having some weak places in the length tested.

It is well known that orientation increases from the periphery to the core of a fiber. This variation in orientation across the fiber width may be considered to be an abnormality of cotton which offsets its strength. This variation depends upon the wall thickness or maturity. In the case of immature fibers (low *C/R*), the cell wall is thin. Hence, the difference in orientation across the cross section of the fiber is small. In mature fibers (high *C/R*), the cell wall thickness is greater. Consequently, the difference in orientation is considerable across the width of the fiber. Because of this difference all the fibrils in mature fibers may not be able to contribute to sustaining the tensile stress together. When the highly oriented inner layers break, the outer layers still have a chance to slip and realign. As a result, the breaking load may level off.[13] When the breaking load is corrected for the linear density in order to calculate the tenacity, the fall at high *C/R* values is even more marked.

For slack mercerized cottons the same trend is observed. It is known that on slack mercerizaion the overall orientation is not changed much.[19] This suggests that the same explanation may also be given for mercerized cottons.

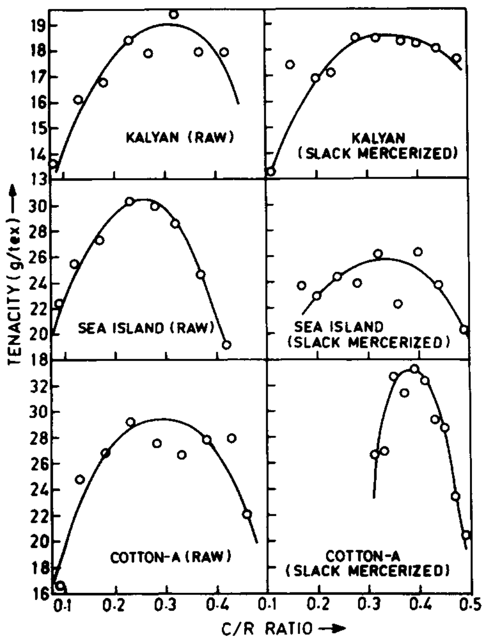


Fig. 7. Relationship between tenacity and C/R ratio for Kalyan, Sea Island, and cotton A: (a) raw cottons; (b) after slack mercerization.

Dependence of Tenacity on C/R Ratio for Raw and Slack Mercerized Cottons

When the tenacity for the raw cotton is plotted against C/R, Figure 7(a), it can be seen that tenacity increases rapidly with C/R, reaches a clearly defined maximum, and then rapidly falls with increasing C/R values. The slopes of the curves also appear to slightly increase with increasing average maturity of the sample. This shape of the tenacity curve with C/R with a clearly defined maximum is to be expected. This is because at high C/R the breaking load tends to level off, whereas the linear density continues to increase. The result of this is that the tenacity decreases rather rapidly at high C/R values.

The effect of slack mercerization on the same cottons can be seen in Figure 7(b). Here, drastic changes in the shapes of the curves from Figure 7(a) will be noted. The changes are more pronounced the higher the average maturity of the initial control cotton. In all cases the tenacity increases with increasing C/R values, reaches a clearly defined maximum, and then decreases at still higher C/R values. However, the position of the maximum shifts to higher C/R values with increasing average maturity.

Dependence of Linear Density on C/R Value for Raw and Slack Mercerized Cottons

From the linear density-versus-C/R curves, Figure 8(a), it can be seen that linear density increases with an increase in C/R. The relationship appears to be curvilinear, while the average slopes of the curves decrease with increasing

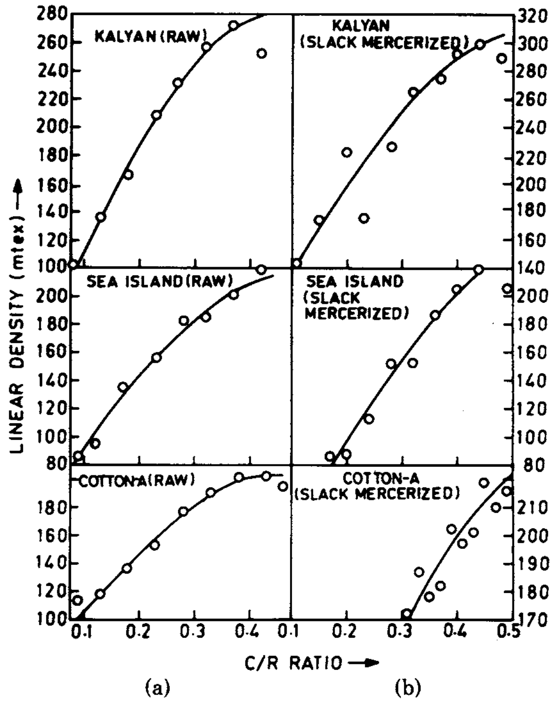


Fig. 8. Relationship between linear density and C/R ratio for Kalyan, Sea Island, and cotton A: (a) raw cottons; (b) after slack mercerization.

average maturity of the cottons determined by the standard method. Possible factors contributing to this type of nonlinear relationship may be described as follows:

Error in Measurement of Ribbon Width. In this study the ribbon width (R) is taken as the maximum width of the fiber between two successive convolutions. Now it is known that the ribbon width changes to some extent with the degree of thickening. According to Lord,[20] the ribbon width first increases as the degree of thickening decreases. With a further decrease in cell wall thickening the edges are curled and hence the ribbon width decreases. This means that at low as well as high maturities the ribbon width will be less than at an intermediate maturity. These factors may contribute to some extent in making the relationship curvilinear.

Effect of Area of Cross Section. The second factor may be the decrease in the cross-sectional area of the inner layers. To explain this, let us consider the fiber to be a perfect cylinder of constant diameter d and of uniform density throughout the cell wall. Now if we divide the cross-sectional area of the cell wall into different layers of equal width l, we find that the outermost layer will have a maximum area, whereas the inner layers will have smaller and smaller areas. Since we have assumed uniform density throughout the cell wall, it is obvious that each layer will weigh less and less as we go from the periphery to the lumen. Theoretically, we have calculated the weights of each layer by considering the density of cotton to be 1.55 g/cm^3. When the weight of n layers is plotted against the ratio nl/d (which is equivalent to C/R), it is found that the graph (not shown here) takes a curvilinear shape. In practice, however, the fiber is neither perfectly cylindrical in shape nor is the diameter of the fiber constant. But since in any variety of cotton the variation in ribbon width is less compared to the variation in the cell wall thickness, the same relationship may hold good between the cross-sectional area of cell wall thickness and the linear density. That means that as the cell wall thickness C increases (i.e., C/R increases), the linear density does not increase in the same proportion.

It should also be remembered that in the present discussion we have considered that the cellulose density is uniform throughout the secondary wall. If the density is not uniform and increases from the perimeter to the lumen, then the effect will be to reduce the curvature depending upon the difference in density. On the other hand, if the density decreases from the perimeter to the lumen, the effect will be to increase the curvature, again depending upon the difference in the density.

The effect of slack mercerization is seen from Figure 8(b). It will be observed that the curves have moved toward higher C/R values. It can also be seen that the curvature has decreased slightly on mercerization. It is known that on mercerization the fiber cross section becomes more regular, hence the variation in ribbon width and maturity (stated as the first factor) will be reduced very much. This will result in a reduction in curvature. However, some residual curvature is still seen, and this may be due to the second factor suggested earlier.

CONCLUSIONS

The following conclusions can be drawn from the results obtained from this study:

1. The observed breaking load frequency data for all cottons can be represented by a β-frequency distribution which is positively skewed. The skewness decreases as the percentage maturity of the cotton sample increases and also as the cotton sample becomes finer. On slack mercerization in 24% (w/w) sodium hydroxide solution, the skewness decreases. This suggests that on mercerization many potentially weak places are strengthened and perhaps some are even completely removed. The parameters m in the β-distribution function is almost constant, whereas n has a tendency to decrease as the maturity increases. It also has a tendency to decrease with increasing fineness.

2. The observed tenacity data for all the cottons can also be represented by β-distributions which are also slightly positively skewed. The skewness has a tendency to decrease as maturity increases, also as the cotton becomes finer. On slack mercerization the frequency distribution of Kalyan and cotton A are not changed much, whereas the frequency distribution of Sea Island cotton becomes more positively skewed. This suggests that Sea Island, which belongs to the *G. barbadense* species, responds somewhat differently to mercerization. As in the case of breaking load, the parameter m is almost constant, whereas n has a tendency to decrease as maturity increases. It also has a tendency to decrease with increasing fineness.

3. The observed frequency data for C/R for raw cottons can also be well represented by β-distributions which are slightly negatively skewed. On slack mercerization, the distributions become highly negatively skewed. The change in the distribution on mercerization is greater the higher the initial maturity of

the raw cotton sample. Contrary to the previous two strength distributions, the parameter m is greater than n, and in this case n remains constant while m has a tendency to change with maturity.

4. The breaking load initially increases with C/R, reaches a maximum between C/R of 0.3 to 0.4, and at higher C/R values the breaking load tends to decrease slightly for all cottons. There does not appear to be much difference with percent maturity. However, on slack mercerization, the curves shift to higher C/R values and the slope also increases with the initial maturity of the control sample. The fall in breaking load at higher C/R values is also more pronounced at higher maturity levels.

5. The breaking tenacity increases rapidly with C/R, reaches a maximum, and then rapidly falls with increasing C/R values. The slope of the curves also appear to slightly increase with increasing maturity. On the other hand, after slack mercerization, drastic changes take place in the shapes of the curves. This is more pronounced the higher the maturity of the initial control cotton. In all cases, the tenacity increases with increasing C/R values, reaches a maximum, and then decreases at still higher C/R values. However, the position of the maximum shifts to higher C/R values with increasing maturity.

6. Linear density is found to increase curvilinearly with increasing C/R. The slopes of these lines reduce with increasing maturity. On slack mercerization, the curves move toward higher C/R values.

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